Abstract

In this paper we discuss the generalization of a problem solved in the thesis dissertation of Rosario Rubio: given an algebraic extension $\mathbb{K}(f_1,\ldots,f_m) \subset \mathbb{K}(x_1,\ldots,x_n)$, compute an intermediate field. If the extension is not algebraic, it is easy to find intermediate fields; our goal is to find one that is algebraic over $\mathbb{K}(f_1,\ldots,f_m)$, which we do by generalizing a theorem by J. Schicho:

Let $A = \mathbb{K}(x_1,\ldots,x_r)$ and $B = \mathbb{K}(y_1,\ldots,y_s)$ be function fields. Let $f, F \in A, g, G \in B$ be non-constant functions. The following are equivalent:

a) There exists $h \in \mathbb{K}(t)$ such that $F = h(f), G = h(g)$.

b) $f - g$ divides $F - G$ in $A \otimes_{\mathbb{K}} B$.

We also study the generalization of these problems to the case where the field $\mathbb{K}(x_1,\ldots,x_n)$ is replaced by the field of fractions of $\mathbb{K}[x_1,\ldots,x_n]/J$ for some prime ideal J.